



11th International Conference on Technology of Plasticity, ICTP 2014, 19-24 October 2014,  
Nagoya Congress Center, Nagoya, Japan

## Determination of optimal conditions for gas forming of aluminum sheets

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### Abstract

This study proposes a method for determining the material constitutive equations and optimal forming conditions on the basis of free bulging tests. The blow-forming tests were carried out at the temperature of 420 °C using aluminum alloy (AMg-6) sheets of a 1 mm thickness. Each test was performed at constant pressure. For each fixed value of the pressure, a series of experiments was carried out with different forming times to obtain evolutions of dome height  $H$  and thickness  $s$ . These data were processed by the proposed method to obtain the flow stress dependence on the effective strain rate. The constitutive equations were obtained by least squares minimization of deviations between the experimental variations of  $H$  and  $s$  and ones predicted by a simplified engineering model. On the basis of the obtained data, the optimum strain rate for AMg-6 processing was determined as one corresponding to the maximum strain rate sensitivity.

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Selection and peer-review under responsibility of the Department of Materials Science and Engineering, Nagoya University

*Keywords:* Bulge forming; Mathematical simulation; Rheological parameters; AMg6; Strain rate sensitivity index; Superplasticity

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## 1. Introduction

Gas forming is an advanced method of complex thin-walled parts production which is used mainly in aerospace industry. To increase the hot forming ability and improve the mechanical properties of the final products, the utilization of superplasticity effect is desired. A computer simulation is necessary to optimize the pressure cycles during the development of superplastic forming technologies. Adequacy of the simulation results depends strongly on the accuracy of initial and boundary conditions and constitutive equations of the material.

The mechanical behavior of superplastic materials is generally described by standard power relation proposed by Backofen et al. (1964):

$$\sigma_e = K \dot{\epsilon}_e^m, \quad (1)$$

where  $K$  and  $m$  are characteristics of the material. The most important characteristic in Eq. (1) is the strain rate sensitivity  $m$ . For superplastic materials the value of  $m$  is greater or equal to 0.3 (Vasin et al., 2000).

Eq. (1) is still the most commonly used constitutive model for simulation of superplastic forming processes. It is a classic power function which forms a straight line with a slope  $m$  if it is plotted in logarithmic scale. At the same time it is well known that a sigmoidal variation of the flow stress with strain rate takes place in the logarithmic scale (Vasin et al., 2000). Thus, Eq. (1) may be applied only as a local approximation describing the sigmoidal curve within a rather narrow range of the strain rates (Enikeev and Kruglov, 1995).

Smirnov (1979) proposed the rheological model of elasto-visco-plastic medium which describes the behavior of superplastic materials in a wide range of strain rates. The constitutive equation corresponding to this model takes the following form:

$$\sigma_e = \sigma_s \frac{\sigma_0 + k_v \dot{\epsilon}_e^{m_v}}{\sigma_s + k_v \dot{\epsilon}_e^{m_v}}, \quad (2)$$

where  $\sigma_0$  is the threshold stress which corresponds to the small strain rates,  $\sigma_s$  is the yield stress at large strain rates,  $k_v$  and  $m_v$  are the parameters of nonlinear viscous element of elasto-visco-plastic medium.

The main advantage of the Smirnov model is its invariance in a wide range of strain rates. At the same time, once the constants  $\sigma_0$ ,  $\sigma_s$ ,  $k_v$  and  $m_v$  are found for a material at a given temperature, the optimal forming conditions can be easily evaluated as a strain rate range corresponding to the required strain rate sensitivity (Aksenov et al., 2013). The effective strain rate sensitivity index  $m$  can be obtained from Eq. (2) as follows:

$$m(\dot{\epsilon}_e) = \frac{d \ln(\sigma_e)}{d \ln(\dot{\epsilon}_e)} = \frac{m_v k_v \dot{\epsilon}_e^{m_v} (\sigma_s - \sigma_0)}{(\sigma_0 + k_v \dot{\epsilon}_e^{m_v})(\sigma_s + k_v \dot{\epsilon}_e^{m_v})}, \quad (3)$$

This function has a peak at the strain rate:

$$\dot{\epsilon}_{\text{opt}} = \left( \frac{\sqrt{\sigma_0 \sigma_s}}{k_v} \right)^{\frac{1}{m_v}}. \quad (4)$$

The determination of material constants on the basis of bulging tests is the focus of many studies. Enikeev and Kruglov (1995) proposed equations for calculation of constants in Eq. (1) on the basis of free bulging tests carried out to a predetermined dome height. Li et al (2004) simulated SPF processes by finite element method (FEM) and applied an inverse analysis to obtain the true material constants in Eq. (1). Giuliano and Franchitti (2007) proposed a method of superplastic material characterization which is able to predict a strain hardening index as well as the constants  $K$  and  $m$ . Giuliano then used this method to produce constitutive equations for Ti-6Al-6V alloy (Giuliano, 2008) and most recently for AA-5083 alloy (Giuliano, 2012).

All of these methods use a Backofen constitutive equation what makes it difficult to determine the optimal forming conditions. In this study a new method is proposed which allows one to determine the constitutive equations in Smirnov form (eqn. 3) as well as in Backofen one (eqn. 2). The proposed method was applied to determine the constitutive equations of AMg6 aluminum alloy. The dependence of strain rate sensitivity index  $m$  on strain rate was analyzed. On the basis of obtained data, the optimum strain rate for AMg-6 processing was determined as one corresponding to the maximum strain rate sensitivity.

## 2. Mathematical model

The scheme of a free bulging process is illustrated at Fig. 1. A metal sheet of initial thickness  $s$  is formed by pressure  $P$  in a die with an aperture radius  $R_0$  and entry radius  $\rho_0$ . At an instant  $t$ , the free part of the dome is assumed to be a spherical surface with radius  $\rho$ .  $H$  is the height of the dome and  $s$  is the current thickness at the dome apex.

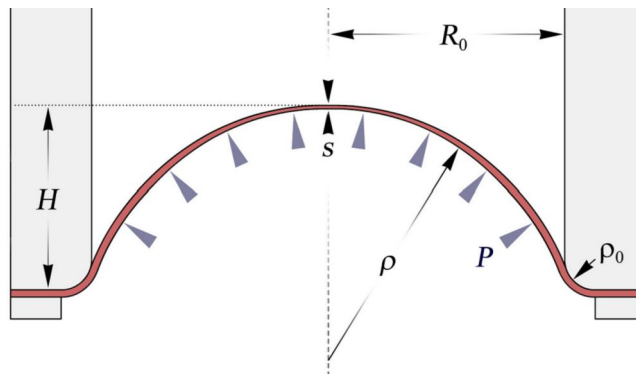


Fig. 1. Scheme of the free bulging process.

By considering the equilibrium conditions of a small element in the dome apex, it is possible to obtain (Kruglov et al., 2002):

$$\sigma_e = \frac{P\rho}{2s}. \quad (5)$$

Equivalent strain and equivalent strain rate in the dome apex can be obtained as:

$$\varepsilon_e = \ln\left(\frac{s_0}{s}\right), \quad (6)$$

$$\dot{\varepsilon}_e = -\frac{1}{s} \frac{ds}{dt} = -\frac{1}{s} \frac{\partial s}{\partial H} \frac{dH}{dt}. \quad (7)$$

The value of  $\rho$  can be expressed as follows:

$$\rho = \frac{(R_0 + \rho_0)^2 + H^2}{2H} - \rho_0. \quad (8)$$

The important point of the mathematical simulation of free bulging process is a relation between the thickness at dome apex  $s$  and its height  $H$ . There are several approaches to express this relation. If experimental data of the dome thickness at different heights are available then the relation  $s(H)$  can be obtained by fitting the experimental data. Aoura et al. (2004) used polynomial fitting of measured apex thickness of domes with different height obtained by tests with constant stress.

In this paper, the approximation of  $s(H)$  dependence was made by integration of the following differential equation:

$$\frac{\partial s}{\partial H} = -\frac{As^\alpha}{\rho}, \quad (9)$$

where  $A$  and  $\alpha$  were constants to be defined. For the combination  $A = 1$ ,  $\alpha = 1$  and  $\rho_0 = 0$ , the solution of eqn. (9) is identical to solution of Jovane (1968) model which assume uniform thickness distribution within the dome surface. For the combination  $A = 2$ ,  $\alpha = 1$ , the solution of eqn. (9) is identical to a relation proposed by Yu-Quan and Jun (1986).

Substituting Eqs. (5), (7) and (9) into Eq. (2) the relation between  $H$  and  $t$  is obtained for Backofen constitutive equation:

$$\frac{dH}{dt} = \frac{\rho}{A} \left( \frac{P\rho}{2K} \right)^{\frac{1}{m}} s^{1-\alpha-\frac{1}{m}}. \quad (10)$$

Substituting Eqs. (5), (7) and (9) into Eq. (1) the relation between  $H$  and  $t$  is obtained for Smirnov constitutive equation:

$$\frac{dH}{dt} = \frac{\rho s^{1-\alpha}}{A} \left( \frac{\sigma_s(P\rho-2s\sigma_0)}{K_V(2s\sigma_s-P\rho)} \right)^{\frac{1}{m_V}}. \quad (11)$$

Effective strain rate  $\dot{\epsilon}_e$  can be determine using Eq. (2) only for effective stress  $\sigma_0 < \sigma_e < \sigma_s$ . Therefore, the eqn. (11) should be restricted as follows:

$$\frac{dH}{dt} = \frac{\rho s^{1-\alpha}}{A} \left( \frac{\sigma_s \max(P\rho-2s\sigma_0, 0)}{K_V \max(2s\sigma_s-P\rho, \delta)} \right)^{\frac{1}{m_V}}, \quad (12)$$

where  $\delta$  is a small value which characterizes a rapid growth of  $H$  at the beginning of the bulging process.

Zero value of  $H$ , according to eqn. (8), leads to an infinite value of  $\rho$ . Therefore, reasonable initial conditions  $H(0) = H_0$  should be applied for both differential Eqs. (10) and (12), where  $H_0$  is a small positive value.

### 3. Experimental and processing techniques

A series of bulge forming tests was performed on the aluminum alloy (AMg6) sheets of a 0.92 mm mean initial thickness, and constant pressures ( $P_j$ ) of  $P_1=0.3$ ,  $P_2=0.35$ ,  $P_3=0.4$ ,  $P_4=0.5$  and  $P_5=0.6$  MPa. The tests were carried out at the temperature of 415 °C with different forming times  $t_i$ . The upper die has cylindrical geometry with the aperture radius  $R_0=50$  mm and die entry radius  $\rho_0=5$  mm. For each fixed value of the pressure, a series of experiments was carried out with different forming times. After the forming, each specimen was measured to obtain the dome height ( $H_i^{exp}$ ) and thickness at the apex ( $s_i^{exp}$ ). Ten tests with different forming times were performed for the pressure  $P_1$ . For the rest of pressures  $P_2 - P_5$  four tests at each pressure were performed.

Thickness measurement was carried out with a micrometer of 0.01 mm divisions. The height of the domes was measured with calipers of 0.05 mm divisions. Obtained data were used to determine the constants of Smirnov and Backofen constitutive equations for AMg6 alloy in two stages.

At the first stage, the values of constants  $A(P_j)$  and  $\alpha(P_j)$  of eqn. (9) were found for every pressure  $P_j$  by nonlinear regression analysis. The eqn. (9) was numerically integrated with the initial condition  $s(0) = s_0$  and initial values of constants  $A(P_j) = 2$  and  $\alpha(P_j) = 1$ . Then, the constants  $A(P_j)$  and  $\alpha(P_j)$  were corrected in order to minimize the objective function  $F_5(P_j)$ :

$$F_s(P_j) = \sum_{i=N_0(P_j)}^{N_1(P_j)} \left( \frac{s_i^{\text{exp}} - s(H_i^{\text{exp}})}{s_i^{\text{exp}}} \right)^2, \quad (13)$$

where  $N_0(P_j)$  is the number of the first test at pressure  $P_j$  and  $N_1(P_j)$  is the number of the last test at pressure  $P_j$ .

At the second stage, the determination of rheological characteristics of the material was performed. The eqn. (10) was used for the Backofen constitutive model and Eq. (12) for the Smirnov one. The material constants ( $K$  and  $m$  for the first model and  $\sigma_0$ ,  $\sigma_s$ ,  $k_v$  and  $m_v$  for the second one) were found by minimization of objective function  $F_{H,s}$ :

$$F_{H,s} = \sum_{i=1}^N \left\{ \left( \frac{H_i^{\text{exp}} - H(t_i)}{H_i^{\text{exp}}} \right)^2 + \left( \frac{s_i^{\text{exp}} - s(t_i)}{s_i^{\text{exp}}} \right)^2 \right\}, \quad (14)$$

where  $N = 26$  is a total number of the experiments.

The objective functions minimization was performed using the method proposed by Nelder and Mead (1965).

#### 4. Results and FEM verification

The rheological constants of Backofen and Smirnov models were determined by proposed technique. The Backofen constitutive model constants were found as:  $K = 155.7$  and  $m = 0.275$ . The Smirnov constitutive model constants were found as:  $\sigma_0 = 12.07$ ,  $\sigma_s = 45.74$ ,  $k_v = 18196$  and  $m_v = 0.929$ . According to Eq. (4), the optimal strain rate for the investigated material is  $\dot{\epsilon}_{\text{opt}} = 7.74 \cdot 10^{-4} \text{ s}^{-1}$ . The maximum strain rate sensitivity value is  $m(\dot{\epsilon}_{\text{opt}}) = 0.298$ .

The obtained constitutive equations were verified by finite element simulation. Two series of FE simulations were performed with the material behavior described by Eq. (1) for the first series and by Eq. (2) for the second one. The simulations were performed in finite element software developed by the authors. Its basic formulation is described in Chumachenko (2005). Axisymmetric viscoplastic three-node triangular elements with mean initial side of 0.3 mm. were used for the simulations. The kinematic boundary conditions were set for the boundary nodes so that they remain fixed after the contact with the dies. On the rest of the bottom boundary a pressure condition was specified.

The  $H(t)$  and normalized  $s(t)$  relations obtained by FEM simulation are compared with the experimental ones in Fig. 2.

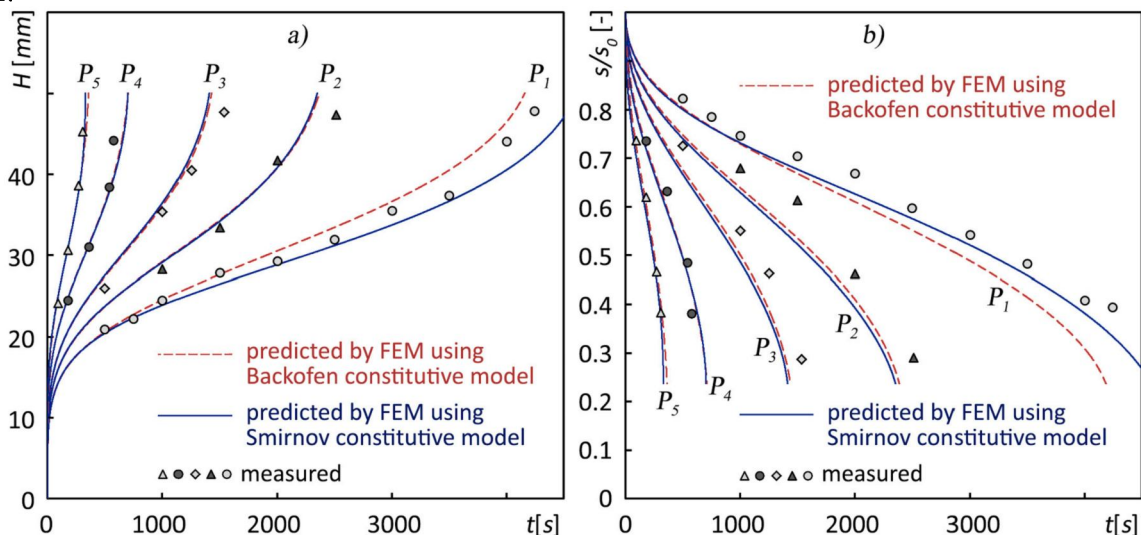


Fig. 2. Evolutions of dome height (a) and thickness (b) predicted by FEM simulations based on Backofen (dashed line) and Smirnov (solid line) constitutive equations compared with measured values.

The greatest differences between the results obtained for Backofen constitutive equation and Smirnov one occur at the pressures  $P_1=0.3$  and  $P_5=0.6$  MPa. At the other pressures the simulation results are almost identical to each other. It can be seen that measured thickness values are generally higher than the ones predicted by FEM. Concerning the prediction of  $H(t)$ , the simulation results are in good correspondence with the experimental ones.

## 5. Conclusions

A free bulging process was considered. Simplified engineering models were used to predict the evaluation of dome height and its apex thickness.

A new technique which enables one to determine the Smirnov constitutive equation constants as well as Backofen ones on the basis of free bulging tests is proposed. The proposed technique consist of two steps. First, the proper approximation is constructed to determine  $H(s)$  relation for each pressure. Then, the material constants are obtained by least square minimization of the deviations between the experimental relations of  $H(t)$  and  $s(t)$  and ones predicted by an engineering model.

The proposed technique was used to determine the constitutive equations of AMg6 aluminum alloy. The obtained material constants were verified by FEM simulation. The optimum strain rate was found for the investigated material to be  $7.74 \cdot 10^{-4} \text{ s}^{-1}$ . At this strain rate it almost reaches the superplastic state, having the strain rate sensitivity index  $m_{max}=0.298$ .

## Acknowledgements

This work includes the results of a project performed under the grant of Russian Scientific Fund "Conducting fundamental scientific research and explorative scientific research by existing teams of scientific laboratories (departments)".

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